

Bertini_real – Numerical Surface Decomposition

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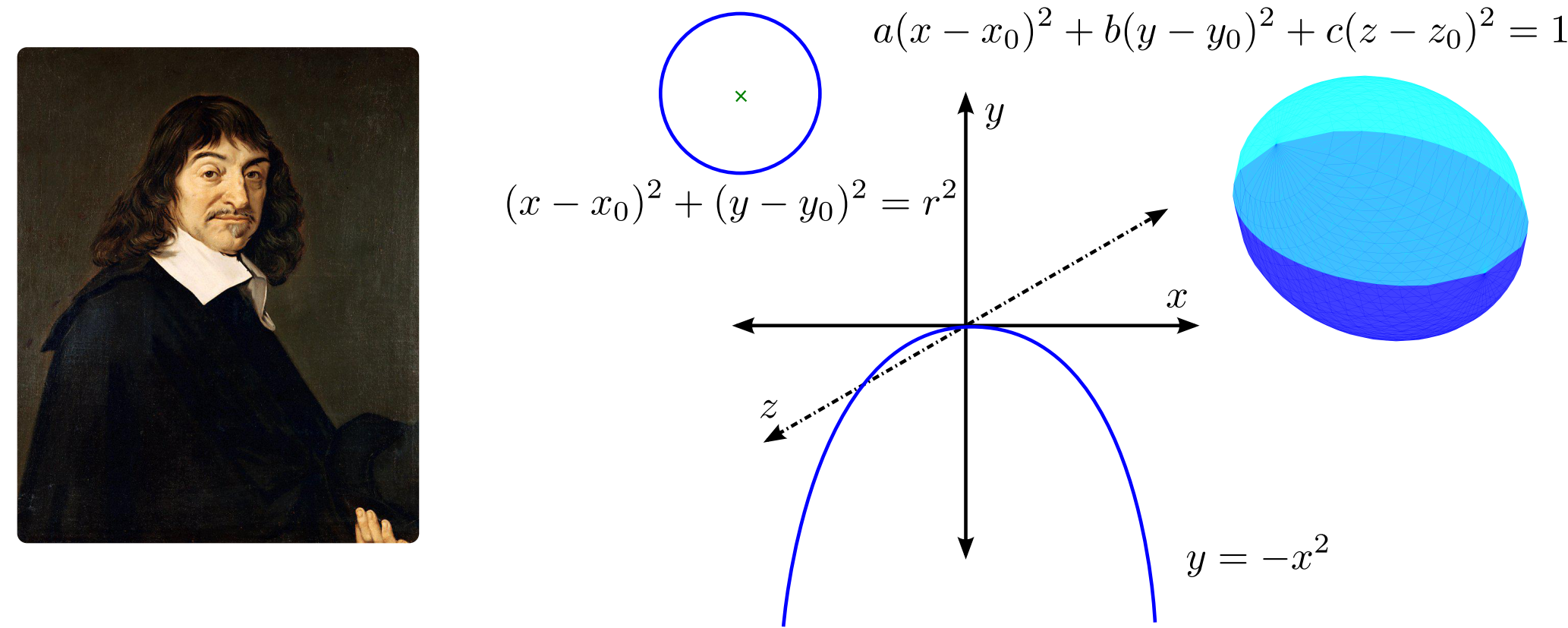
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Background

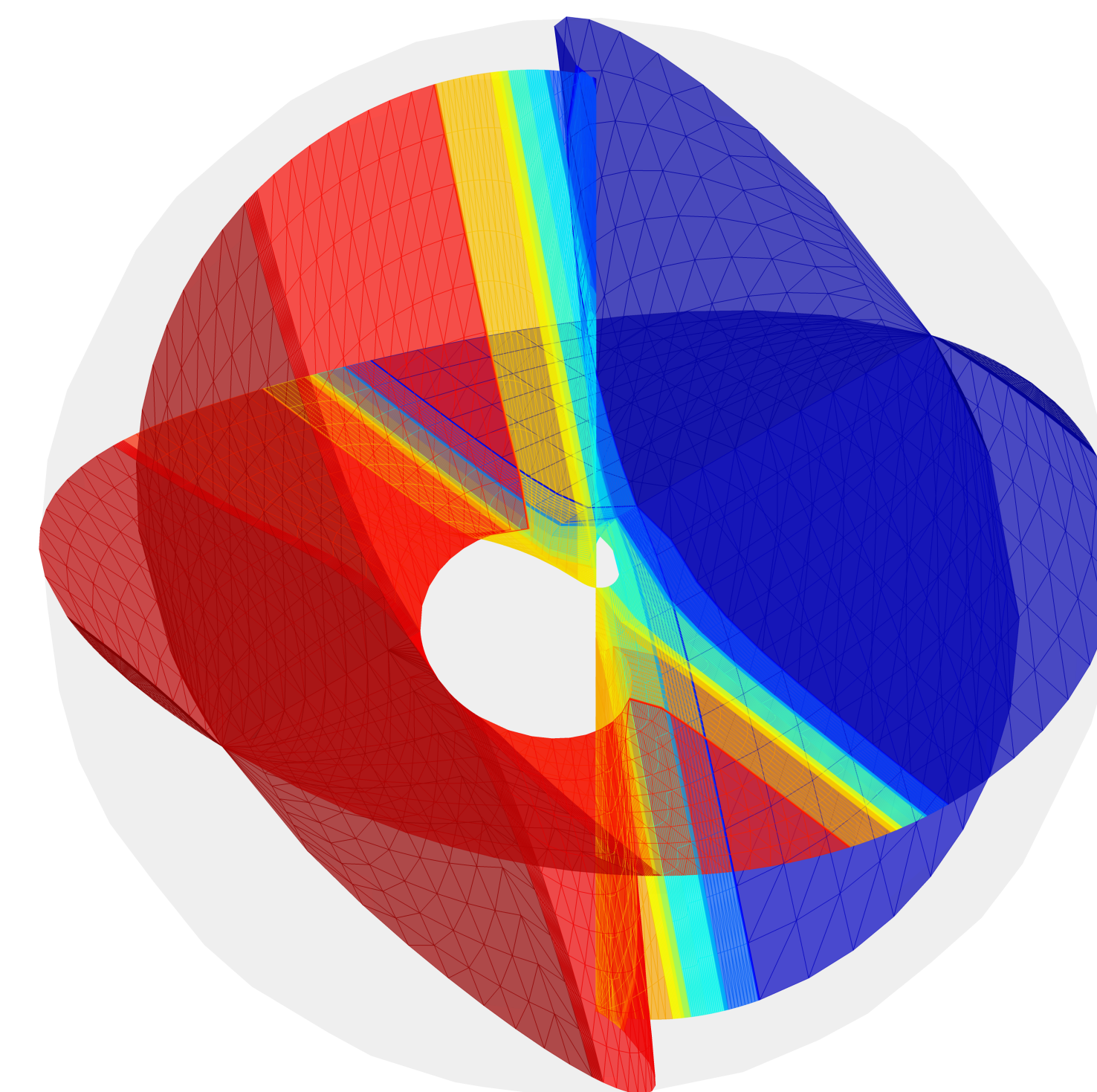
René Descartes (1596-1650) connected algebra and geometry.



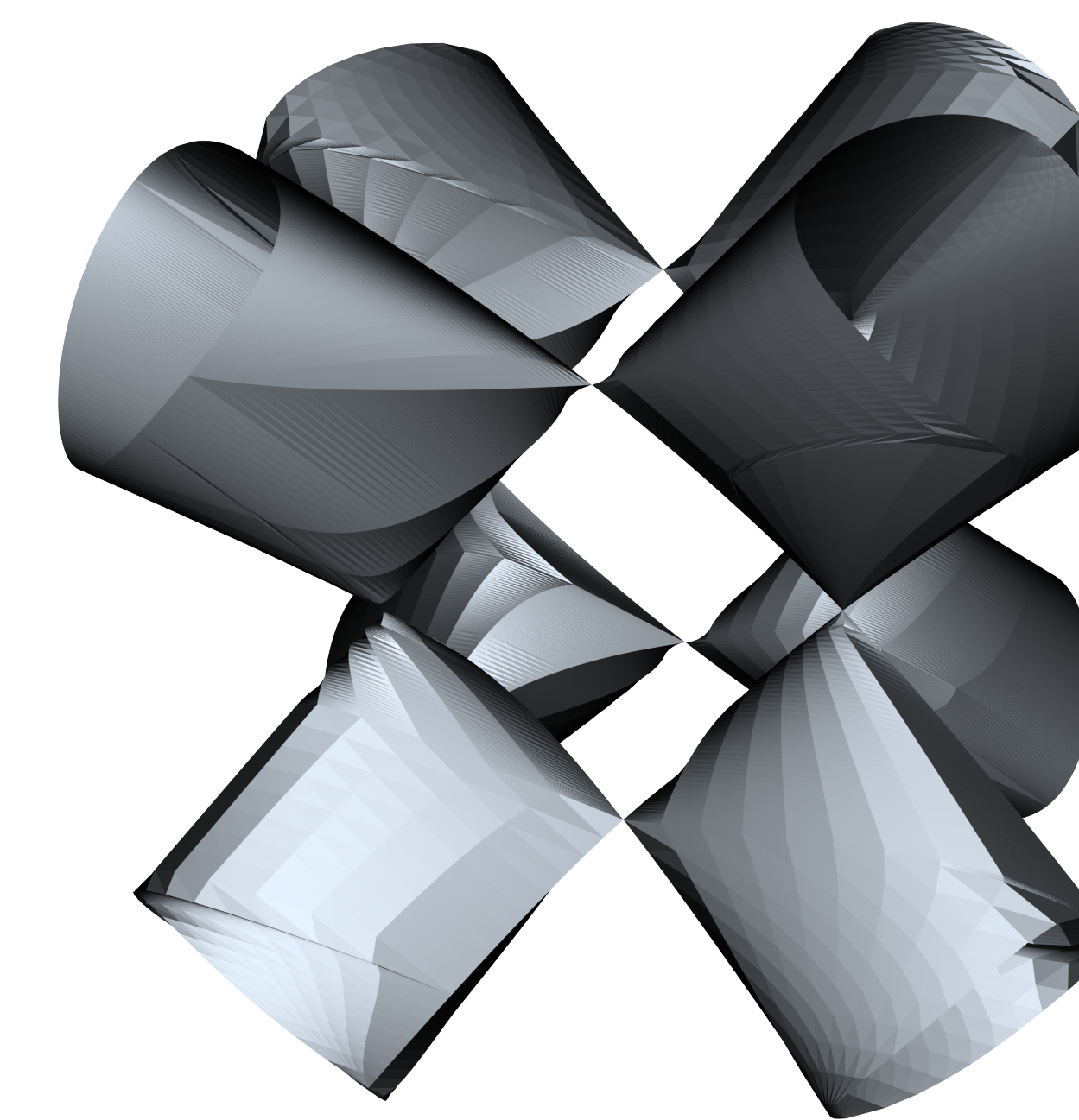
As a computer program, Bertini_real seeks to:

- ✓ Connect modern algebra and geometry,
- ✓ Push the boundaries of 3D printing,
- Enable implicit optimization and integration.

Surface Decomposition



solitude⁴
 $x^2yz + xy^2 + y^3 + y^3z - x^2z^2 = 0$



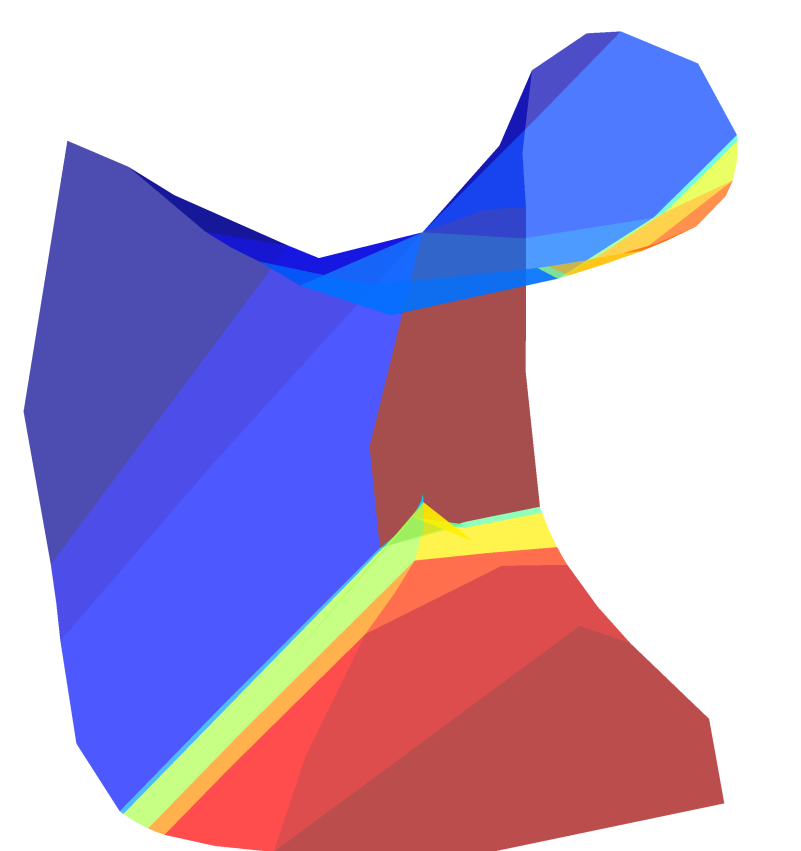
kummer⁵
 $x^4 + y^4 + z^4 - (y^2z^2 + z^2x^2 + x^2y^2) - (x^2 + y^2 + z^2) + 1 = 0$

3D Printing

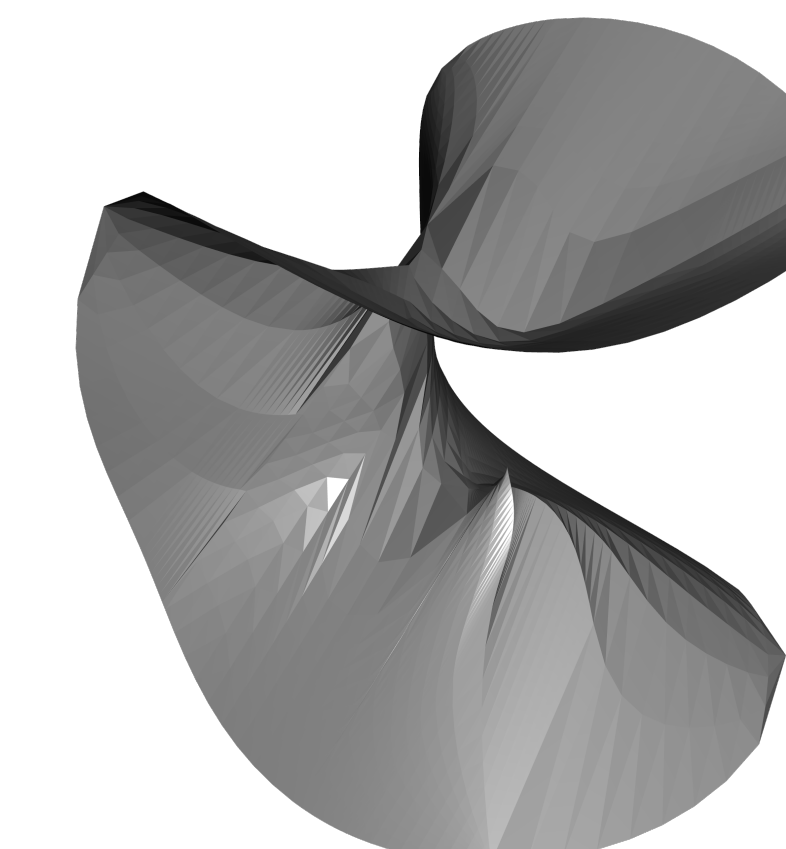
1. Run Bertini

```
***** Witness Set Decomposition *****
| dimension | components | classified
|-----|-----|-----|
| 2         | 1         | 4
|-----|-----|-----|
***** Decomposition by Degree *****
Dimension 2: 1 classified component
degree 4: 1 component
*****
```

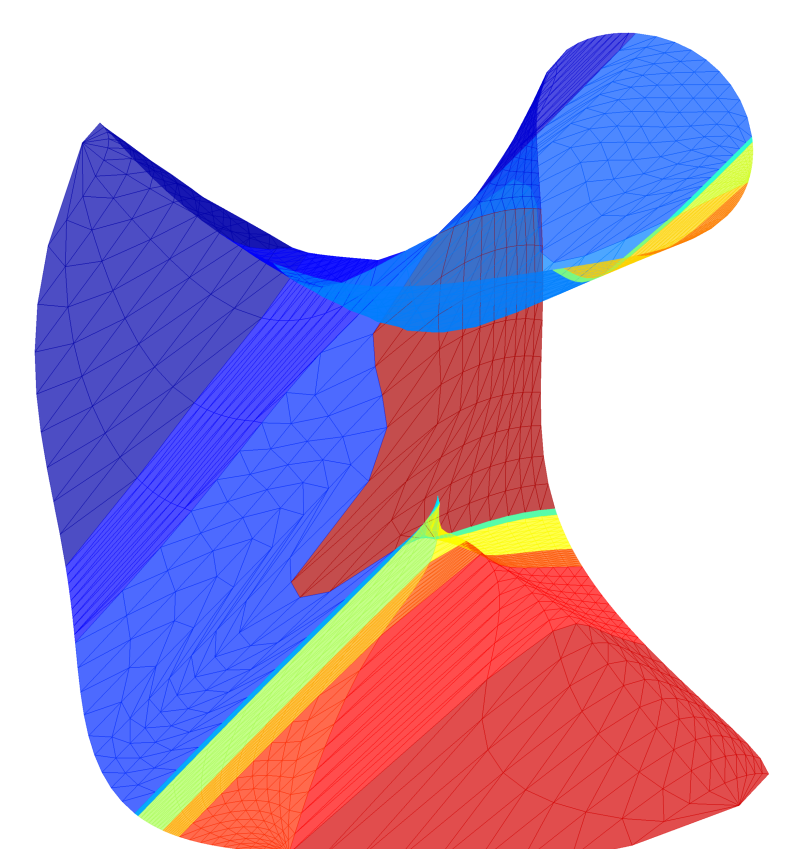
2. Run Bertini_real



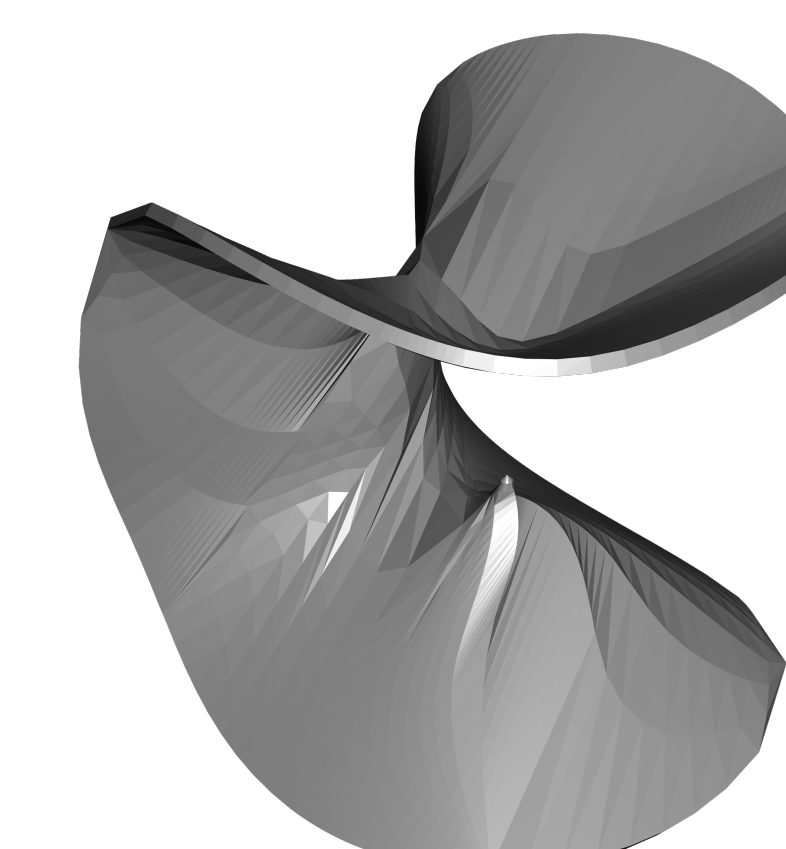
4. Process into .stl



3. Refine



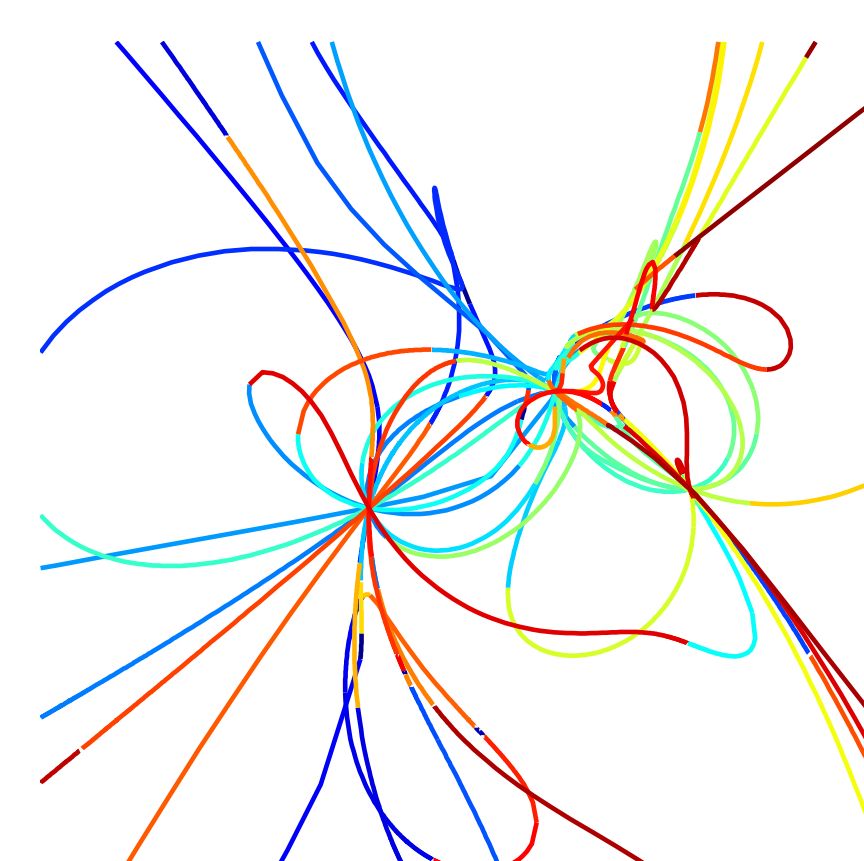
5. Thicken surface



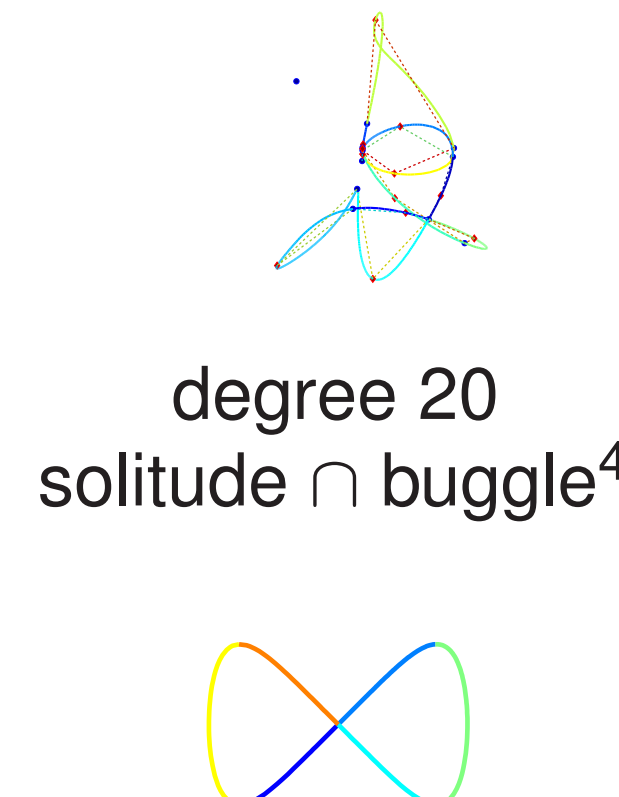
6. Print



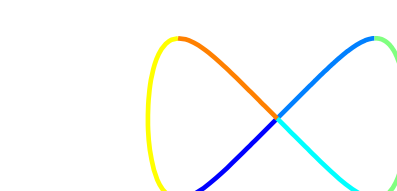
Method – Curves¹



3-3 Burmester curve.³
degree 630, 14 variables.



degree 20
solitude $\cap$ buggle⁴



butterfly curve
 $x^6 + y^6 = x^2$

1. Find critical points

2. Intersect with sphere

3. Slice

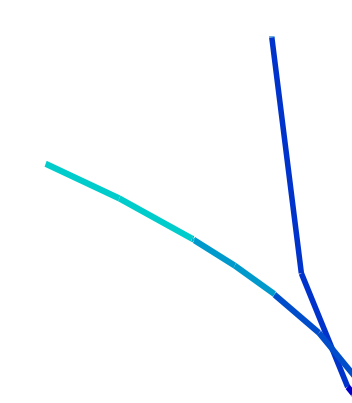
6. Refine

5. Merge

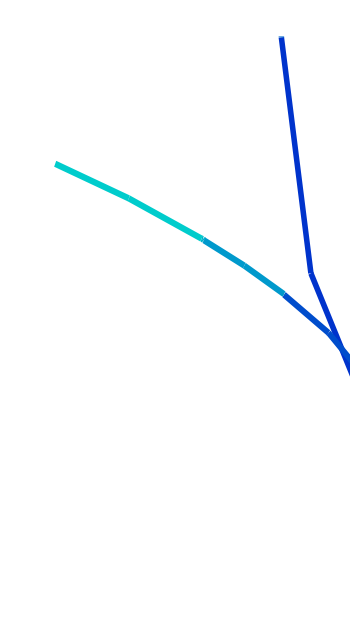
4. Connect the dots

Method – Surfaces²

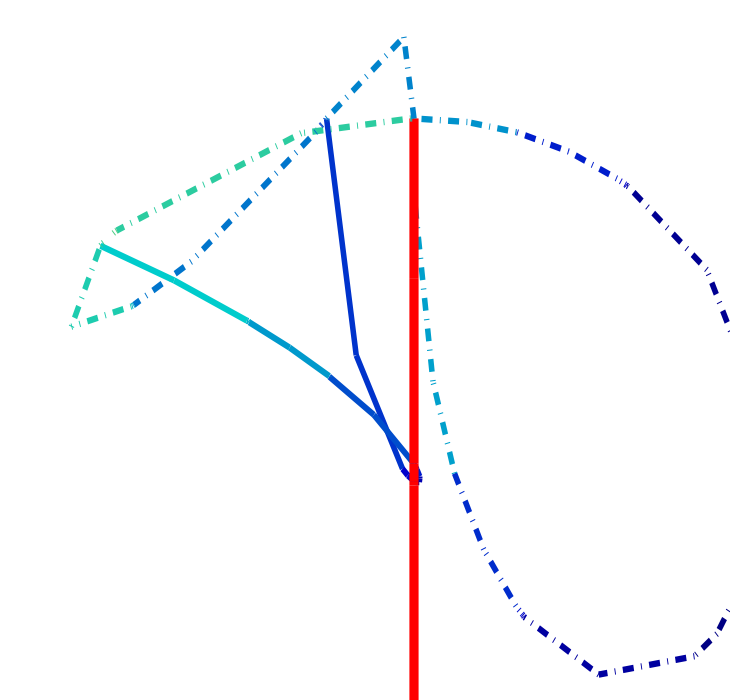
1. Decompose
critical curve



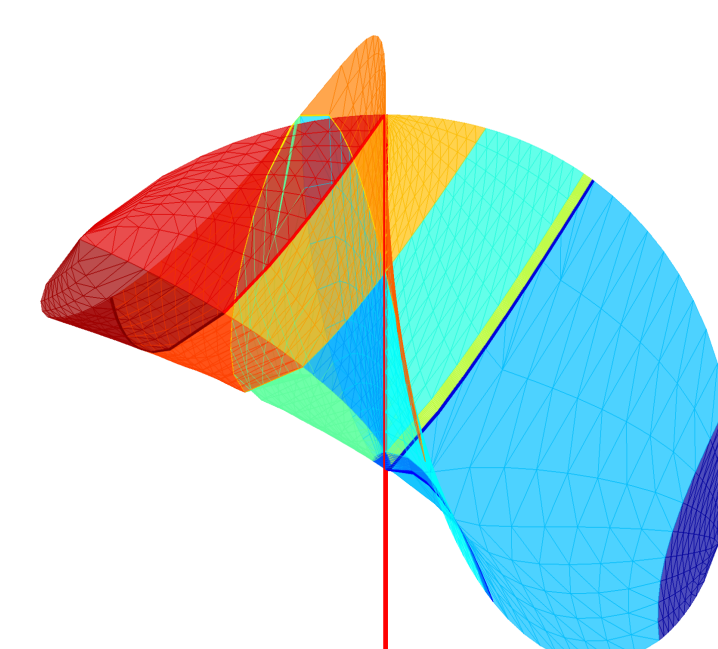
2. Decompose
singular curves



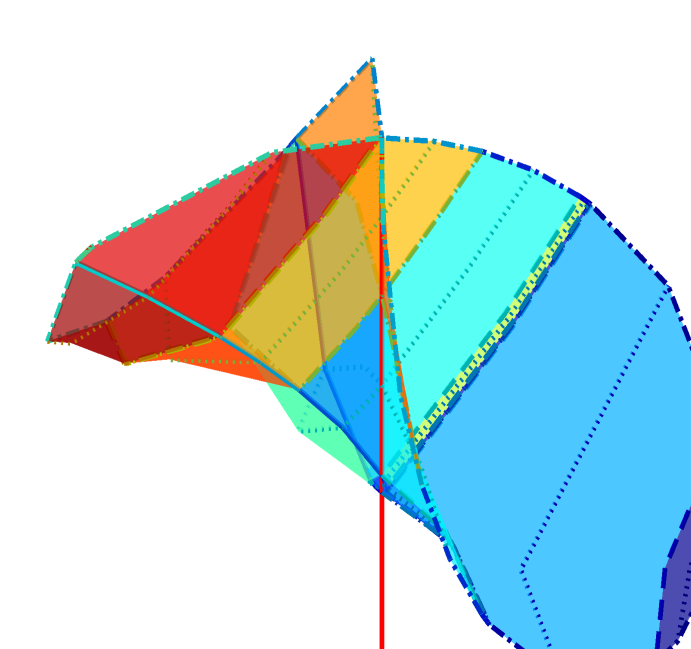
3. Intersect with
sphere



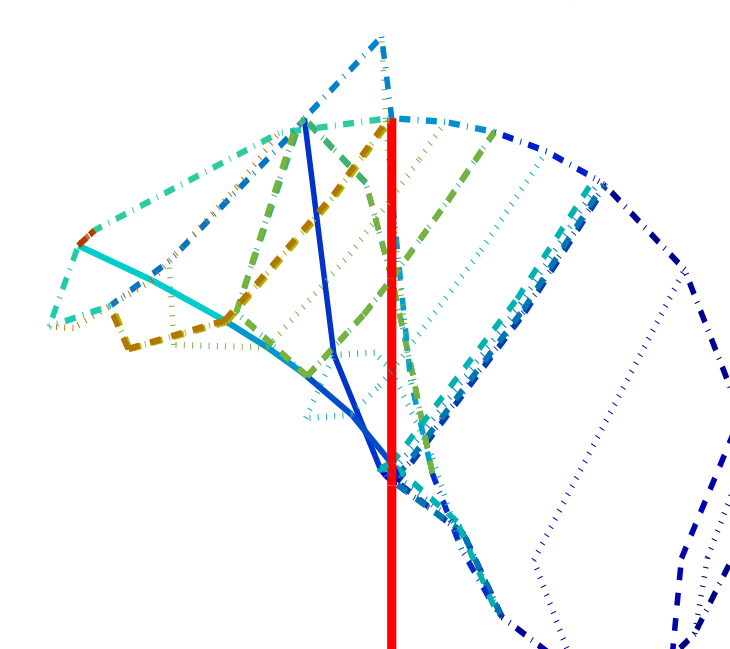
6. Refine



5. Connect the dots



4. Slice



References & Acknowledgements

- ¹ Lu, et al. "Finding all real points of a complex curve". *Cont. Math.* 448:183. 2007.
- ² Besana, et al. "Cell decomposition of almost smooth real algebraic surfaces". *Numer. Algo.* 63:645-678. 2013.
- ³ Tong, et al. "Four-bar linkage synthesis for a combination of motion and path-point generation". *IDETC/CIE 2013*.
- ⁴ Algebraic Surface Gallery. www1-c703.uibk.ac.at/mathematik/project/bildergalerie/gallery.html
- ⁵ Fischer. *Mathematical Models*. Friedrich Vieweg & Son. 1986.
- ⁶ Bates, et al. "Bertini: Software for Numerical Algebraic Geometry". Available at bertini.nd.edu
- ⁷ Hauenstein, Wampler. "Isosingular Sets and Deflation". *Found. Comp. Math.* 13(3):371-403. 2013.

- Bertini_real relies entirely on the excellent homotopy continuation solver Bertini,⁶ and uses *isosingular deflation*⁷ to track on singular curves.
- This project received generous support from DARPA YFA.