

Printing Algebraic Geometry

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May 24, 2016



Outline

Overview

Algebraic Geometric models

Math model $\rightarrow$ printable model

Favorite pieces

Applications

Components of a 3d printing setup

- ▶ Printer
- ▶ Materials
- ▶ Software
- ▶ Models

Printers

additive

- ▶ Stereolithography – light-curing a liquid
- ▶ Fused filament fabrication – layer deposition (what I use)
- ▶ Sintering – laser-fusing a powder
- ▶ Exotics and specialty – welding, jetting

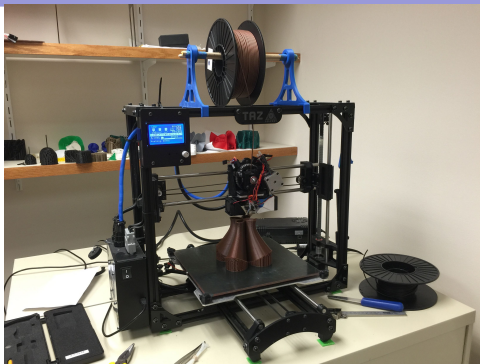
subtractive

- ▶ CNC machine
- ▶ Router, lathe, other machining

hybrid

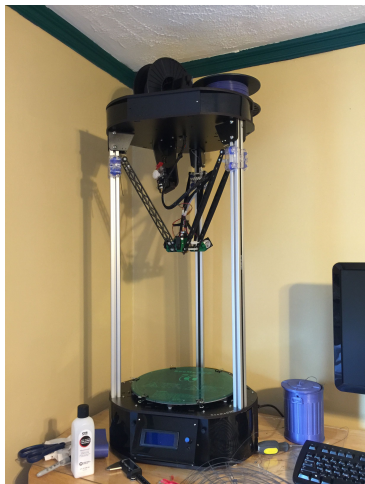
- ▶ Combine subtractive and additive printing into one machine
- ▶ experimental, expensive

Jon's Taz4



- ▶ Cartesian robot
- ▶ Pre-assembled
- ▶ \$2000
- ▶ Direct drive extruder
- ▶ open source hardware/software

My Rostock



- ▶ Delta robot
- ▶ Built from kit
- ▶ \$1000
- ▶ Bowden extruder
- ▶ open source hardware, software

Materials

Many materials for 3d printing exist. Examples:

- ▶ Rigid plastics – PLA, ABS, Nylon, Taulman Tglase, PETG
- ▶ Flexible – Ninjaflex, Semiflex
tires, otherwise fragile things
- ▶ Dissolvable – HIPS, PVA
use as support material
- ▶ Conductive
- ▶ Exotics – wood, carbon fiber, resin-metal

Software

Some piece of software has to generate machine instructions for the printer. Model not fed directly to printer!

- ▶ A slicer
Input model, output text file `gcode` interpreted by machine.
 - ▶ Slic3r
 - ▶ Cura
 - ▶ Simplify3d
 - ▶ And many others
- ▶ Plater / control software
an interface for controlling the machine before prints, and prepping
 - ▶ Matter control
 - ▶ Repetier Host
 - ▶ Many others

Models

To print, you have to have something to print!

How to get the model?

- ▶ Use someone else's model. Thingiverse, YouImagine, 123D Gallery, Shapeways
- ▶ Design. Solidworks, Sketchup, Rhino3D, Blender. So many!
- ▶ Program. OpenSCAD (with CGAL under the hood!), and others.
- ▶ Compute. Mathematica exports directly to `.stl`. Matlab Exchange has `stlwrite` for `fv` structs. I wrote `Bertini_real`, which computes algebraic surfaces.

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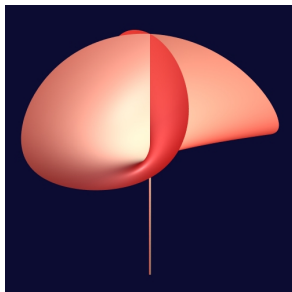
Applications

Equation $\rightarrow$ print

To print the algebraic geometry, start from equations

Example: Whitney umbrella

$$x^2 - y^2z = 0$$



<http://homepage.univie.ac.at/herwig.hauser/bildergalerie/gallery/whitney.jpg>

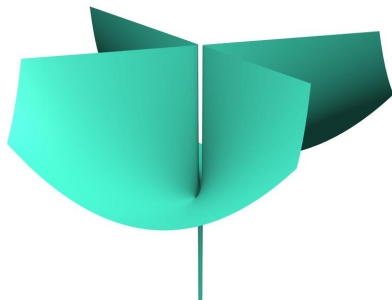
Previous image is image, not model. Can't 3d print it!
Ok, let's summon an isosurface plotter. Come, Matlab!

```
function whitney_matlab

[y,x,z] = ndgrid(linspace(-2,2,100),...
                linspace(-2,2,100),...
                linspace(-2,2,100));
f = x.^2-y.^2.*z;
isosurface(x,y,z,f,.001);
view(3);
camlight
axis equal

end
```

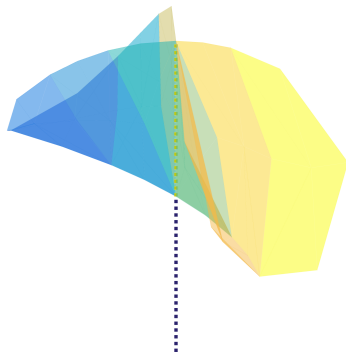

Whitney – isosurf



problem – line singularity $x = y = 0$ doesn't resolve properly!

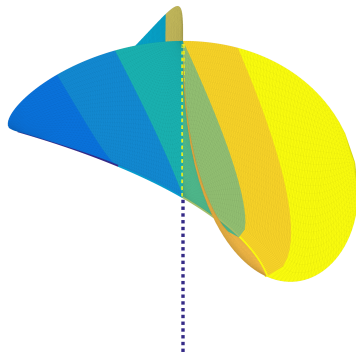
Numerical Cellular Decomposition

Use Bertini_real to perform Cellular Decomposition of real part
of complex degree 3 surface



Rough $\rightarrow$ smooth

Smoothed, with Bertini_real sampler. Now we have a model!



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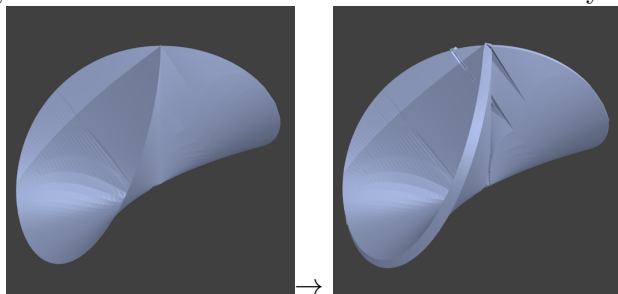
Favorite pieces

Applications

Zero extent models

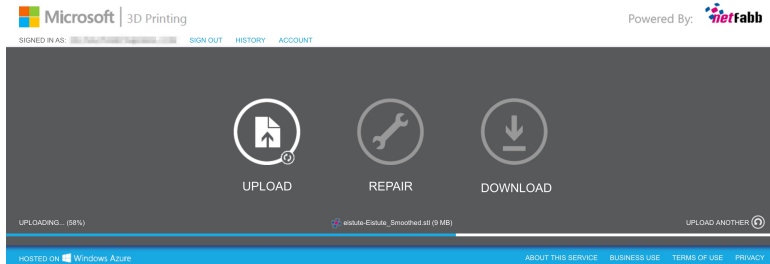
The majority of surfaces I come across are not closed, or at least a portion isn't.

This means they are not natively printable. To give them extent, the best tool I have found is Blender's solidify routine.



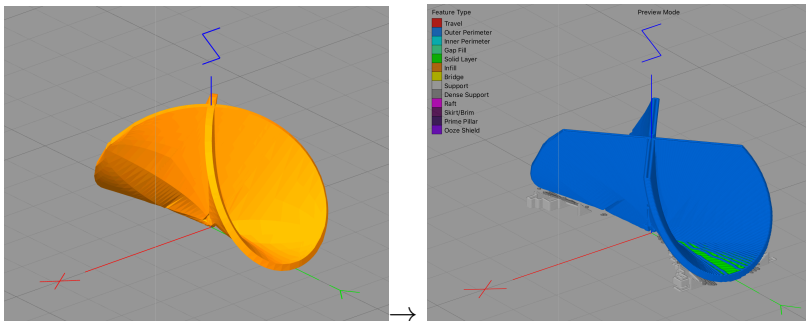
Internal geometry and self-intersections

The slicer is particular. It requires perfect geometry. To get closer to perfect geometry, I feed my models to a fixer, particularly `netfabb.azurewebsites.net/`. It's the best free solution I've found.



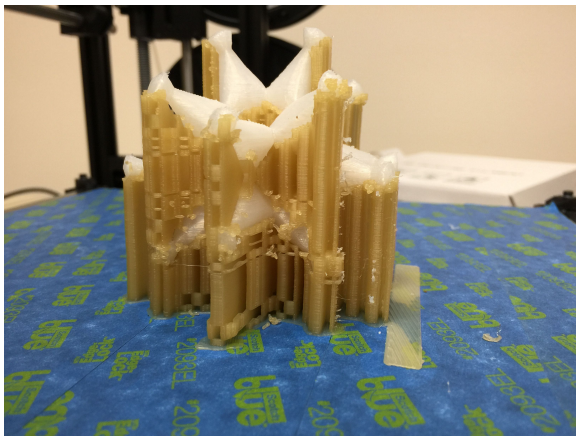
Hopefully at a printable model now

Now can try to slice the model.



A note on support

Overhangs and mid-air portions of an object are challenging. Support helps us print otherwise impossible things.



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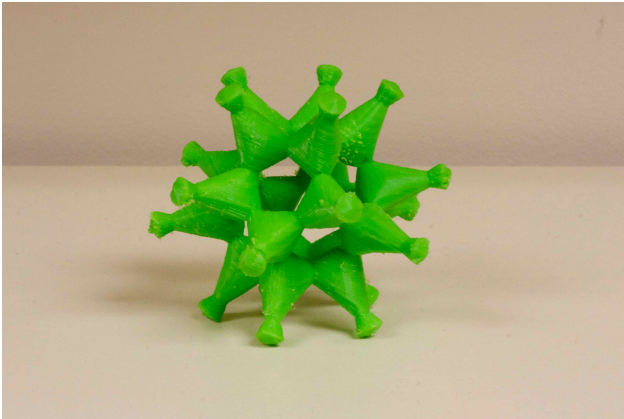
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Applications

Barth Sextic

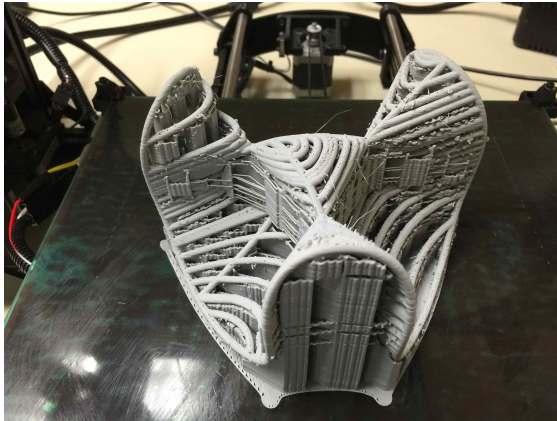


Two-color clebsch cubic



27 real lines
made for Bernd Sturmfels

Cayley nodal cubic – skeleton



support inseparable from object :(

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A common question about math – Why?

Happy to answer!

- ▶ Teaching
- ▶ Visualization == understanding
- ▶ Possible industrial or scientific applications

Teaching – Topology



Wireframe Klein Bottle by Gnietschow

Wireframe Klein Bottle by
Gnietschow
Published on November 2, 2014
www.101.org/movies/000783.jpg/157567



Attribution-ShareAlike
BY SA



Summary

1. Run Bertini

Compute $\mathbb{C}$ witness points

```
***** Witness Set Decomposition *****
| dimension | components | classified | unclassified |
|-----|-----|-----|-----|
| 2 | 1 | 6 | 0 |
***** Decomposition by Degree *****
Dimension 2: 1 classified component
degree 6: 1 component
```

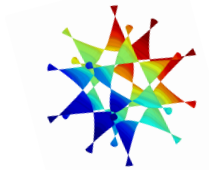
2. Run Bertini_real

Solve many $\mathbb{R}$
polynomial systems



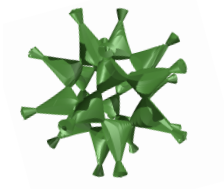
3. Refine

Smooth blocky regions
face-by-face



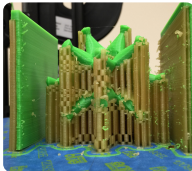
4. Post-process

Solidify, save as .stl



5. Print

Ninjabflex, PVA support



6. Remove support

Warm water bath



Thanks for your kind attention