Workspace Multiplicity and Fault Tolerance of Cooperating Robots

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D. Brake partially supported by grants NSF-DMS-09087551 and NSF-IIS-0812437.
D. Bates partially supported by grants NSF DMS–0914674.
V. Putkaradze partially supported by grant NSF-DMS-09087551.
A. Maciejewski partially supported by grant NSF-IIS-0812437.

November 12, 2015

Scenario

Many robots are designed to work in remote or dangerous locations. Suppose:

- System has multiple robotic arms,
- Workspaces of arms overlap,
- Arms capable of grasping each other,
- One arm undergoes free-swinging joint failure. [1]

Then, to maximize a function of the pre- and post-failure workspaces:

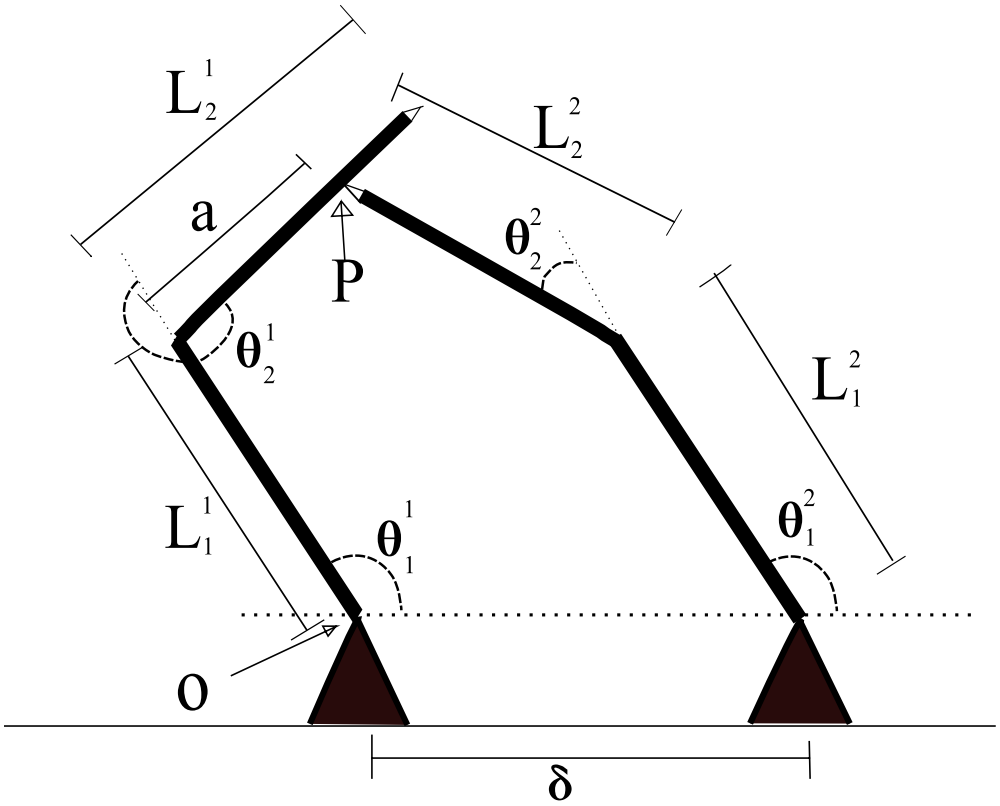
1. How close should the arms be?
2. Where should they connect post-failure?

Our method to answer the questions is:

- Use *homotopy continuation* to solve inverse kinematics at a random sample of space.

[1] English, Maciejewski, IEEE Trans. Robot. Automat., 1998.

The Problem



2D Example of problem.

Optimize to determine a and δ:

- Robots separated by distance δ,
- Right robot (#2) grasps left (#1) at point P, at distance a from the second joint of robot 1,

so that some function of the post-failure workspace is maximized.

The Problem

There are four workspaces to consider:

- W_1, W_2, the *pre-failure* workspaces for robots 1 & 2.
- $W_\cap = W_1 \cap W_2$, the *intersection* of the two pre-failure workspaces. Depends on δ.
- W_f, the *post-failure* workspace. Depends on δ and a.

Our objective is to maximize some function Ω of these four workspaces. We used,

$$\Omega = (1 - \lambda) \cdot |W_f| + \lambda(|W_1| - |W_\cap|),$$

where λ is a problem-specific weighting factor.
Note: $|W_1| - |W_\cap|$ is a measure of the benefit of having the second robot.

Workspace measure

We define a multiplicity-weighted workspace measure,

$$|W| = \int_W m(x)\mathrm{d}x$$

- $m(x)$ is the number of configurations placing the robot in a position x in the workspace.
- If $x \notin W$, then $m(x) = 0$.
- Since the set of singular points is a set of measure zero, they don't affect this number.

Jacobian Method

Traditionally, one would use *Jacobian Method* to solve problem:

- Start robot from initial state, use Jacobian Control[2] to move robot to desired point.:

$$\dot{x} = J(\theta) \cdot \dot{\theta}$$

- If robot can come within ϵ of point, it is in workspace. Else, out.
- Alternately, one could move robot to boundary of workspace, and move in *nullspace* of Jacobian to trace boundary.

Method has shortcomings, however:

- Can only find *one* solution per point.
- Can miss voids in workspace.
- Hard to measure volume of calculated workspace.

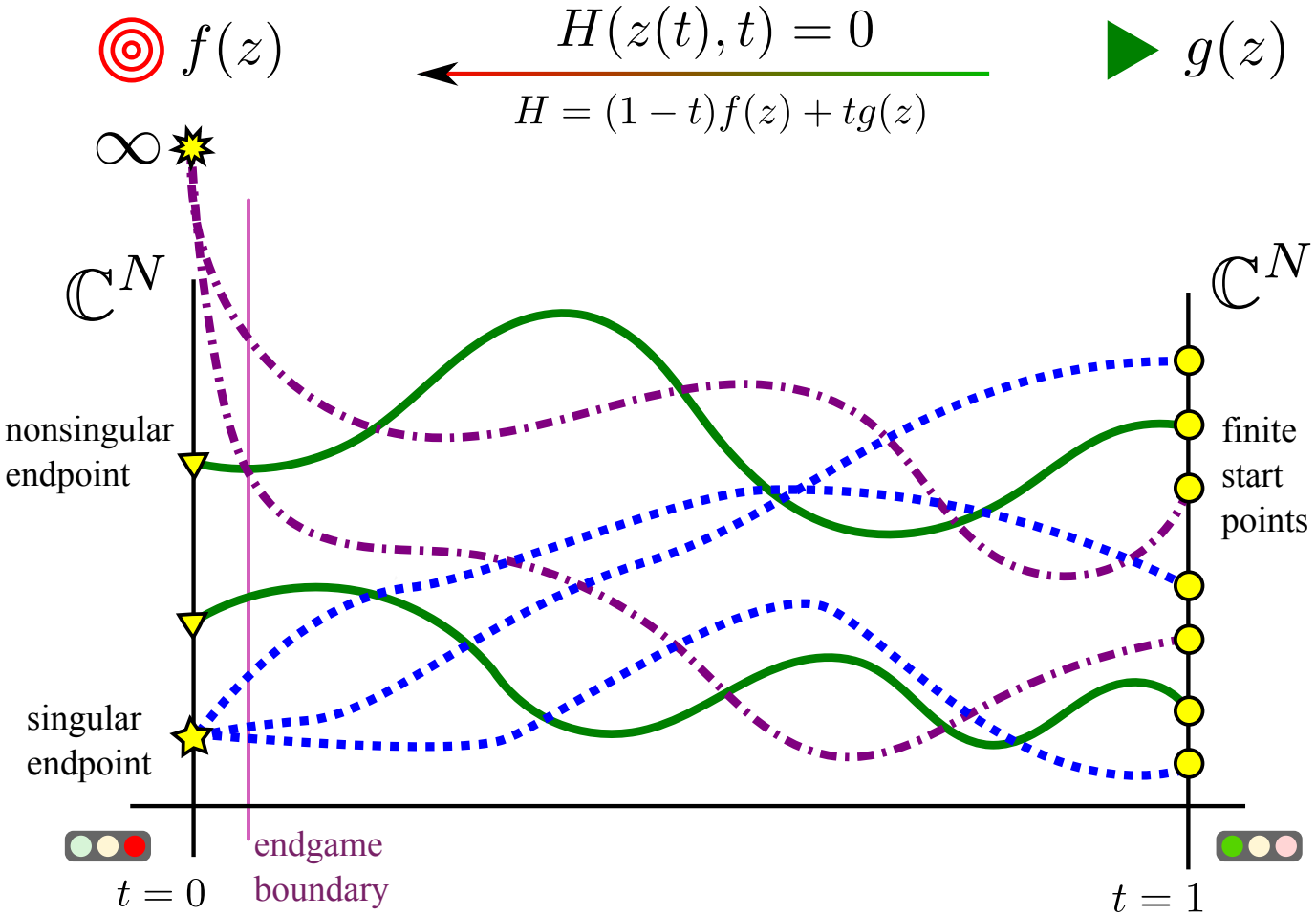
[2] Fu, Gonzalez, Lee. *Robotics : Control, Sensing, Vision, and Intelligence.* New York: McGraw-Hill, 1987

How to compute $m(x)$

We solve the inverse kinematics equations using *homotopy continuation*.

- Inverse kinematics equations are polynomial systems.
- Algebraic Geometry (AG) is the study of solving polynomial systems.
- Numerical AG mature enough to solve large systems numerically with high accuracy, quickly.
- We use the freely available package *Bertini*.
- Full 6D inverse equations have no closed form solution - solve numerically.
- Finds *all* isolated solutions.

Homotopy Continuation



Bertini and Paramotopy

The *Bertini* [3] software package is a sophisticated tool for solving polynomial systems with homotopy continuation. Some benefits and features include:

- Arbitrary and adaptive precision,
- Parameter and custom homotopy definitions,
- Positive-dimensional component detection and (complex) sampling,
- Parallel version with MPI,
- Free to use,
- New version Bertini2 under development, with many additions and improvements.

To compute the workspaces, we use *Paramotopy* [4], which lives on top of Bertini.

[3] Bates, Hauenstein, Sommese, Wampler. 2015. Available: http://bertini.nd.edu

[4] Brake, Bates, Niemerg. paramotopy.com

Optimization method

Input : DH parameters for each robot, λ,
samplings of δ, a, S
Output: Optimal $\delta_{\max}, a_{\max}$, $\Omega_\lambda(\delta_{\max}, a_{\max})$

1 Compute $W_1 = \{x \in S \mid m(x) > 0\}$;
2 Compute $W_\cap(\delta)$ using W_1;
3 Compute $W_f(\delta, a)$ using W_1;
4 Evaluate $\Omega_\lambda(\delta, a)$;
5 Find $\delta_{\max}, a_{\max}$ maximizing Ω_λ;
6 **return** Maximizers $\delta_{\max}, a_{\max}$ and value $\Omega_\lambda(\delta_{\max}, a_{\max})$;

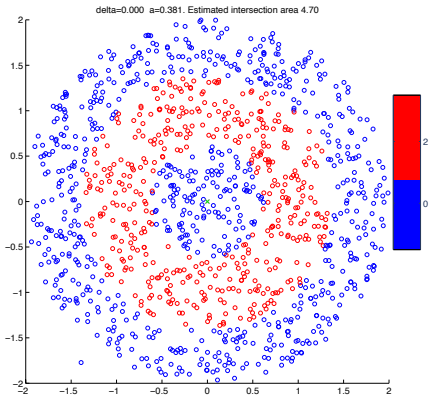
Monte Carlo Estimation

- For each of the four workspaces,
  - W_1, W_2, the prefailure independent workspaces,
  - $W_\cap$, the intersection of the prefailure workspaces, and
  - W_f, the cooperative postfailure workspace.

  we randomly sample space *guaranteed to contain* the workspace.

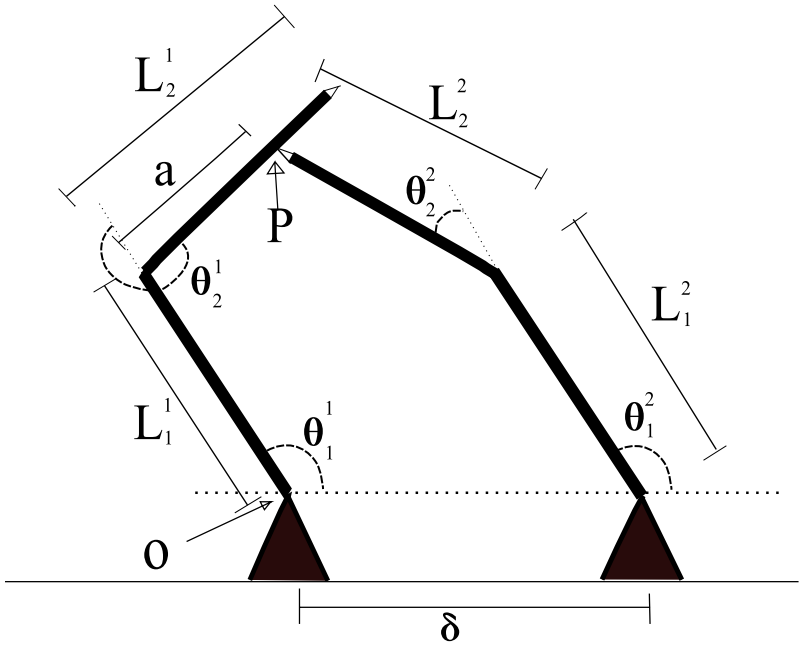
$$|W| = \lim_{r\to\infty} |S| \frac{1}{r} \sum_{j=1}^{r} m(x_j), \qquad (1)$$

where $|S|$ is the typical Euclidean measure of the sampling space.



2-Joint Planar Robot, Step 1

- Both robots identical, with all links of length 1. Then $W_1 = W_2$, translated by δ.
- Initial sampling of $[-2\,,2] \times [-2\,,2]$ for computing W_i.
- Inverse kinematics equations is a system of two equations in two variables (2 by 2), which we transform into a 4 by 4 system:



$$0 = \begin{cases} c_1 c_2 - s_1 s_2 + c_1 - x \\ s_1 c_2 + c_1 s_2 + s_1 - y \end{cases} . \tag{2}$$

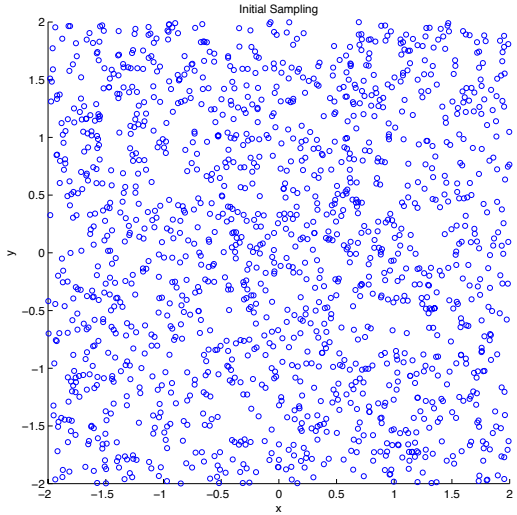
These equations are augmented by trigonometric identities,

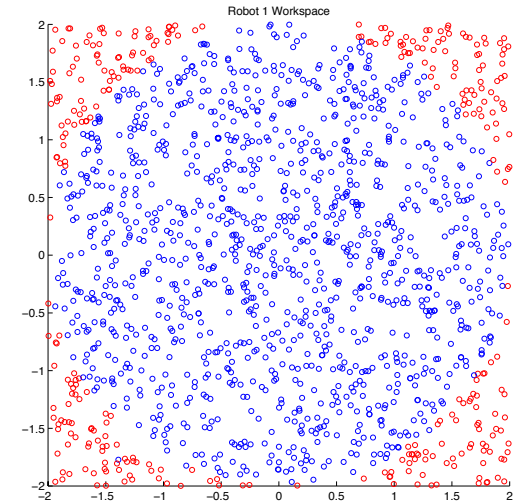
$$c_i^2 + s_i^2 - 1 = 0, \quad i = 1, 2. \tag{3}$$

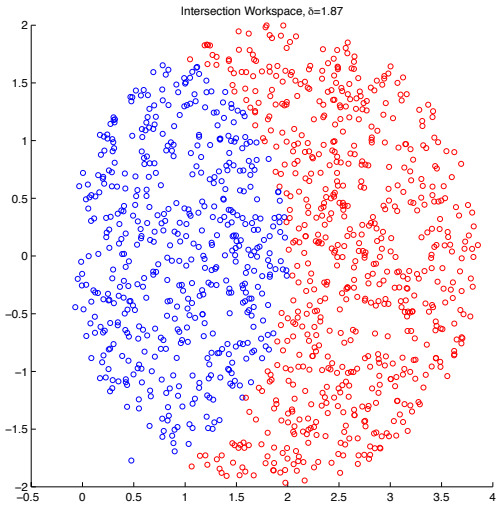
2-Joint Planar Robot, Step 2

Having W_1, next we compute $W_\cap$.

- Since $W_\cap \subset W_1$, throw out points from original sample that had no solutions.
- $W_\cap = W_\cap(\delta)$, so translate sample for each δ.
- Use same equations, solve with respect to robot 2.

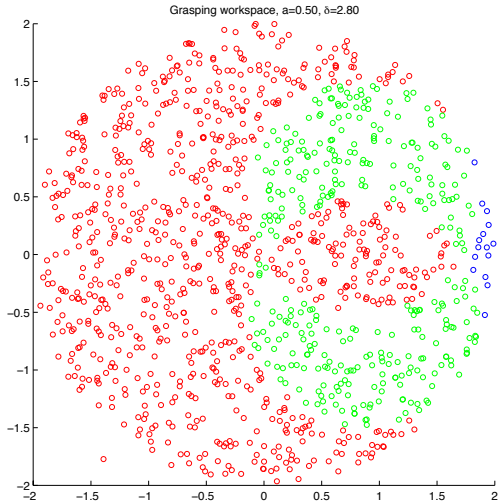






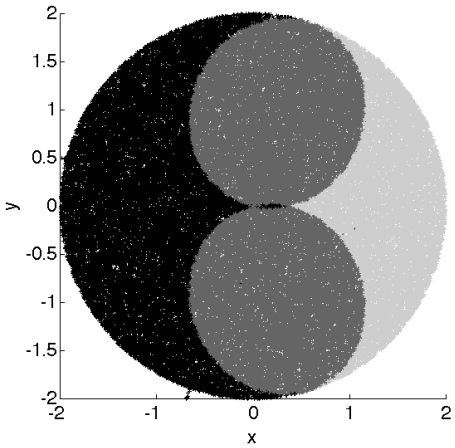
2-Joint Planar Robot, Step 3

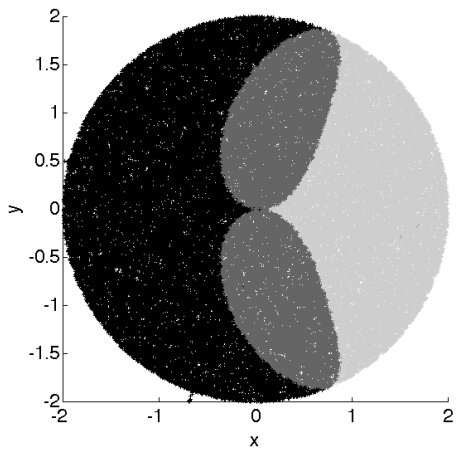
- Using $W_f \subset W_1$, solve new equations for grasping configuration.
- Solve for each pair of (δ, a) values.
- W_f system is 8 by 8:

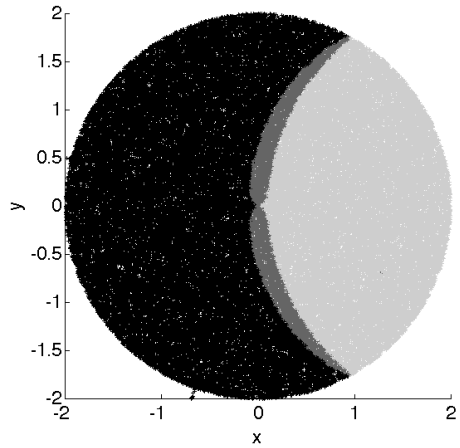


$$0 = \begin{cases} c_1 c_2 - s_1 s_2 + c_1 - x \\ s_1 c_2 + c_1 s_2 + s_1 - y \\ a c_1 c_2 - a s_1 s_2 + c_1 - (c_3 c_4 - s_3 s_4 + c_3 + \delta) \\ a s_1 c_2 + a c_1 s_2 + s_1 - (s_3 c_4 + c_3 s_4 + s_3) \\ s_i^2 + c_i^2 - 1 \end{cases}$$

W_F Examples





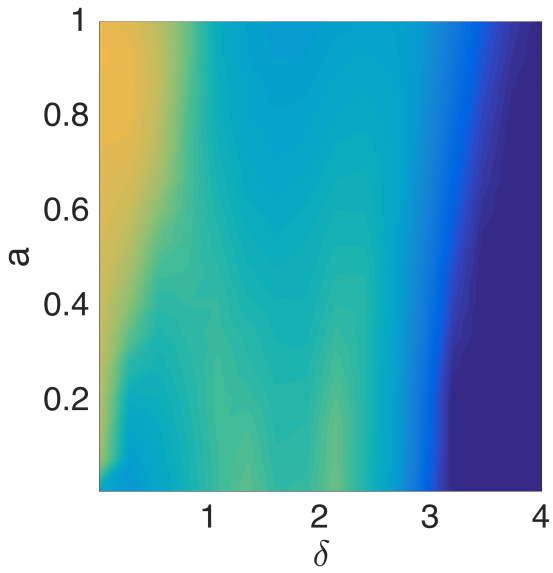


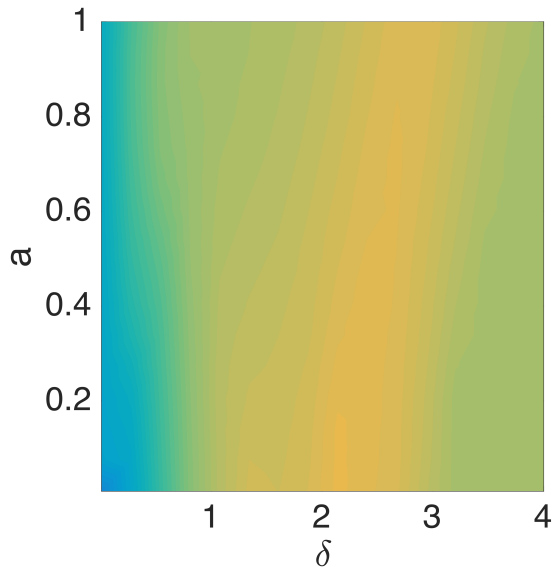
Cooperative workspaces for $\delta = 2$, $a = 0.1, 0.5, 0.9$.

Black: unreachable by robot 1 in cooperative mode, dark gray : two configurations reachable, light gray: four configurations reachable. White area is outside the workspace of robot 1.

$\Omega = (1-\lambda)\cdot|W_f| + \lambda(|W_1| - |W_\cap|)$

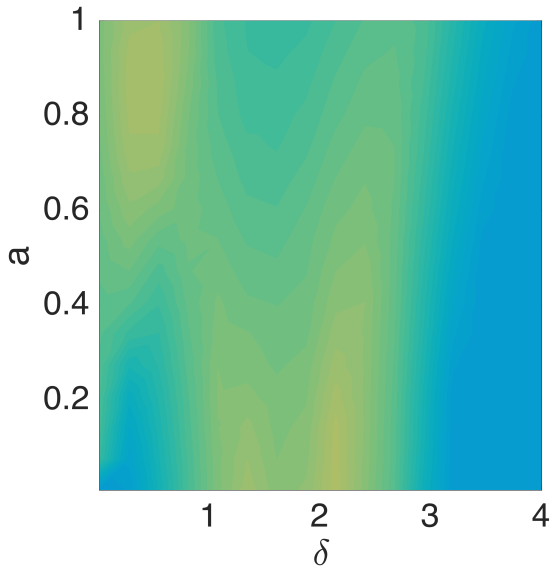
$\lambda = 0.$

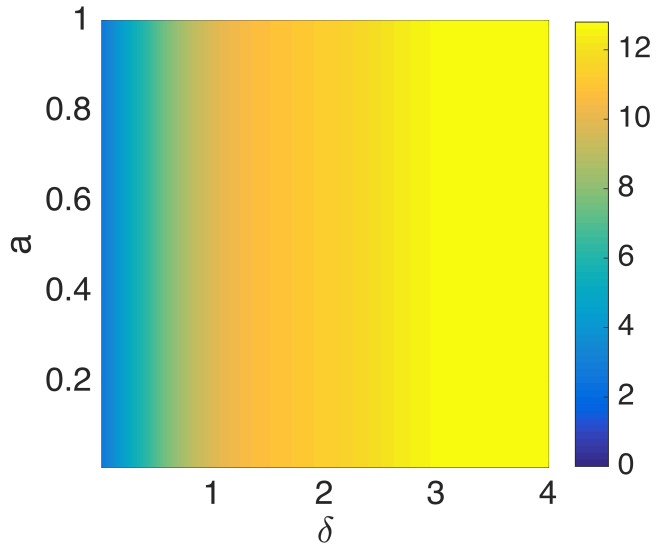




$\lambda = 2/3.$

$\lambda = 1/3.$



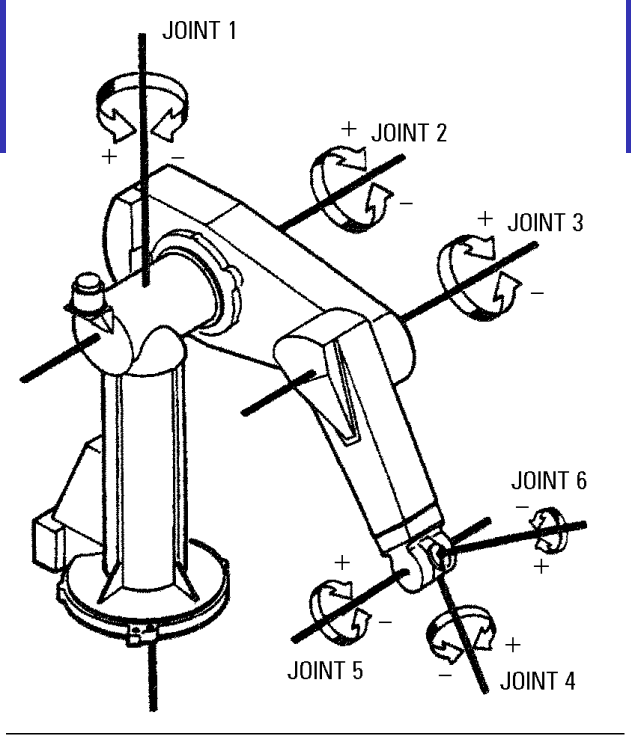


$\lambda = 1.$

Puma 5xx Robot

A fully 6D robot.

- First three joints control position,
- Last three, a wrist, controls orientation.



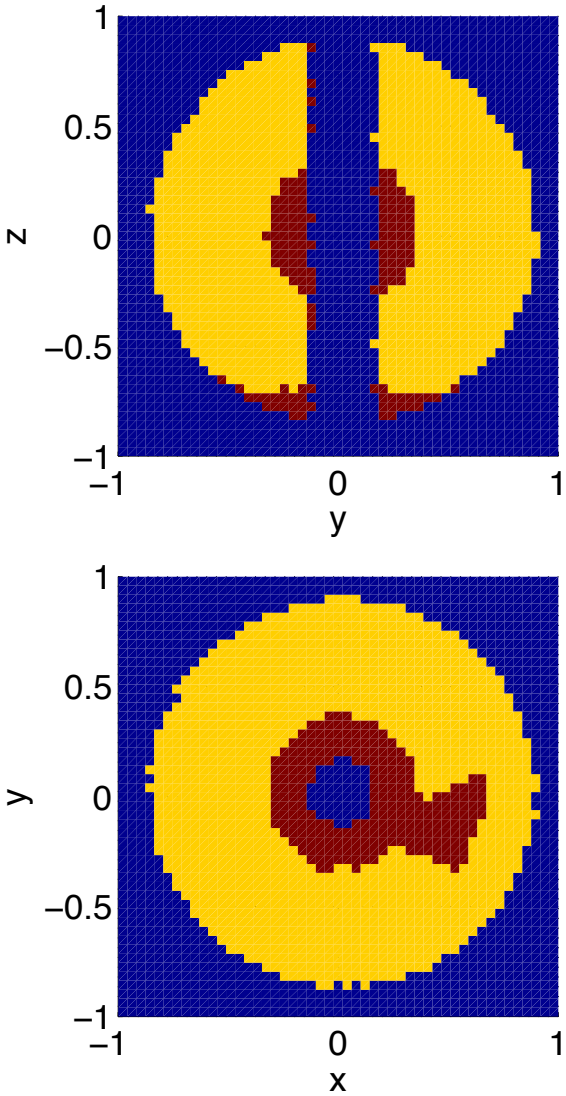
PUMA Denavit Hartenberg Parameters [5] , first three joints					
Joint i	θ_i	α_i	a_{i-1}	d_i	Type
1	θ_1	$-\pi/2$	0	243.5mm	R
2	θ_2	0	431.8 mm	-93.4 mm	R
3	θ_3	$\pi/2$	-20.32 mm	433.1mm	R

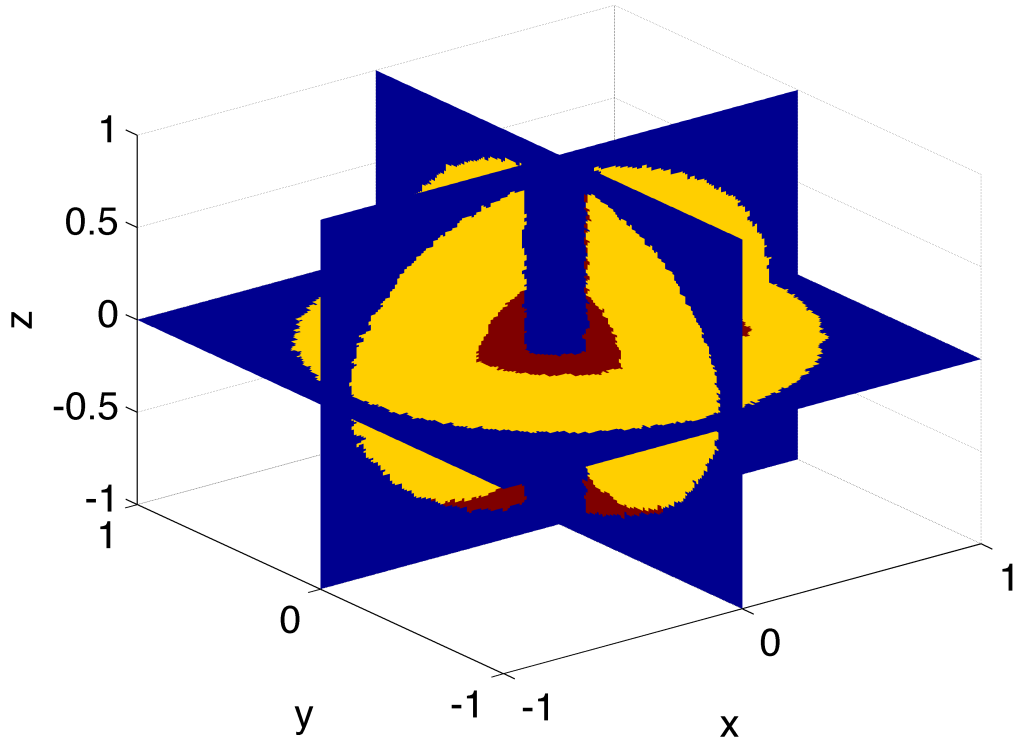
[5] Corke, Armstrong-Helouvry. Robotics and Automation. 1994

3D W_i Equations for Puma 500

$$0 = \begin{cases} 0.15005s_1 + 0.4318c_1c_2 + 0.4318c_1c_2c_3 - 0.4318c_1s_2s_3 - x \\ 0.4318c_2s_1 - 0.15005c_1 + 0.4318c_2c_3s_1 - 0.4318s_1s_2s_3 - y \\ 0.4318s_2 + 0.4318c_2s_3 + 0.4318c_3s_2 + 0.67 - z \\ s_i^2 + c_i^2 - 1 \end{cases}$$

One equation per dimension, plus the Pythagorean identity relating the sine and cosine pairs, for each $i = 1..3$.

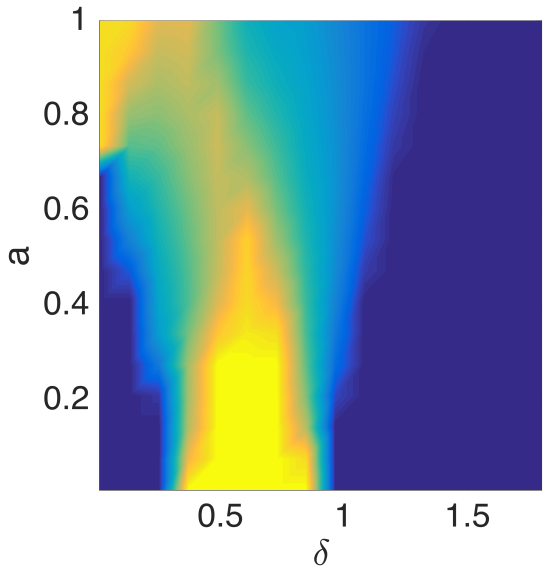


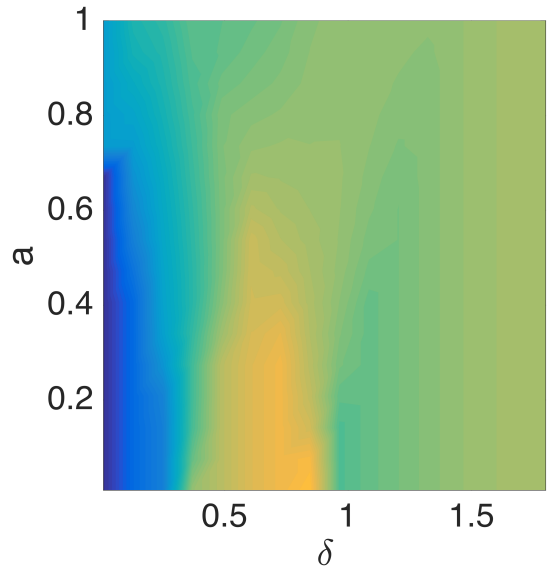


Left: Slices of the pre-failure PUMA workspace W_1 by the planes $x = 0$ (top) and $z = 0$ (bottom). Right: Combined picture of workspace with three slices $x = 0$, $y = 0$, $z = 0$. Yellow color: areas accessible the angles satisfying joint limits. Red color: real solutions violating joint limits. Dark blue region: inaccessible.

Objective Ω

$\lambda = 0.$





$\lambda = 2/3.$

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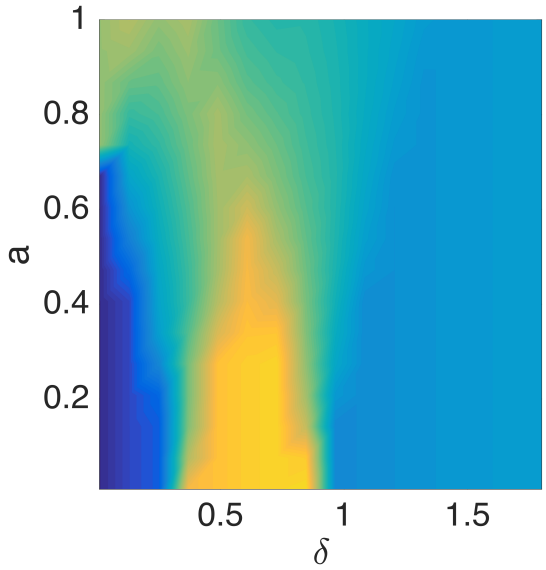

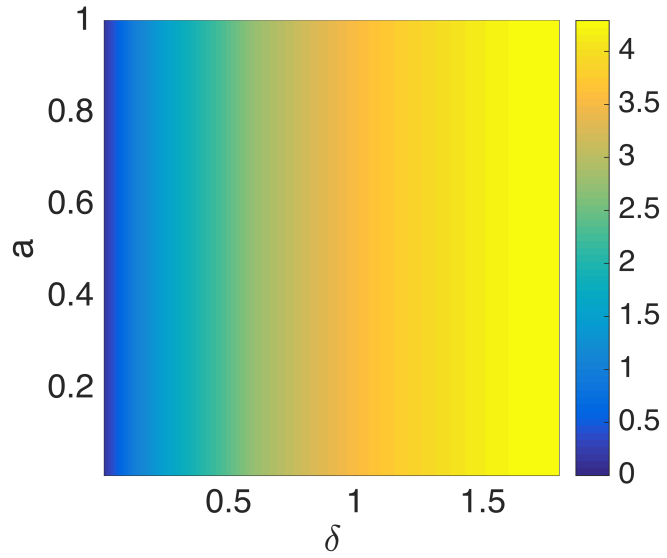

$\lambda = 1.$

Conclusion and outlook

- Inverse Kinematics equations may be turned into polynomial systems, and solved via homotopy continuation.
- Robot workspaces may be estimated via Monte Carlo methods - random sampling, and repeated solving of the inverse kinematics.
- Optimal placement of robot bases, and sockets on links, depends on Ω, which in turn depends on user's choice of weighting factor λ.
- Generalization to 6D robots will require uniform sampling of 3D rotations.