Illustration of numerical algebraic methods for workspace estimation of cooperating robots after joint failure

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Scenario

Many robots are designed to work in remote or dangerous locations. Suppose:

- System has multiple robotic arms,
- Workspace of arms overlaps,

What if one arm undergoes free-swinging joint failure?
If we grasp the broken arm with the functional, in order to restore workspace,

1. how close should the arms be?
2. where should they connect post-failure?

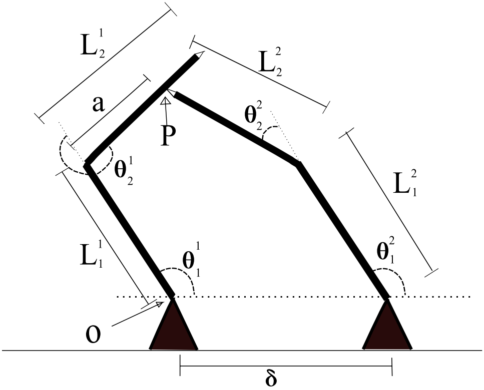
Scenario

1. How close?
2. Where should they connect?

Answer to these two questions depends on how we measure the workspaces. Regardless of measure, our method to answer the questions is the same:

- Use *homotopy continuation* to solve inverse kinematics at a randomly chosen sample of space (monte carlo workspace estimation),
- Maximize an objective function Ω to find optimal placement and grasping location.

The Problem



2D Example of problem.

Optimize to determine a and δ:

- Robots separated by distance δ,
- Right robot (#2) grasps left (#1) at point P, at distance a from the second joint of robot 1,

so that workspace is maximized in some sense.

The Problem

There are four workspaces to consider:

- W_1, W_2, the *pre-failure* workspaces for robots 1 & 2.
- $W_\cap = W_1 \cap W_2$, the intersection of the two pre-failure workspaces. Depends on separation δ.
- W_f, the *post-failure* workspace. Depends on separation δ and grasping parameter a.

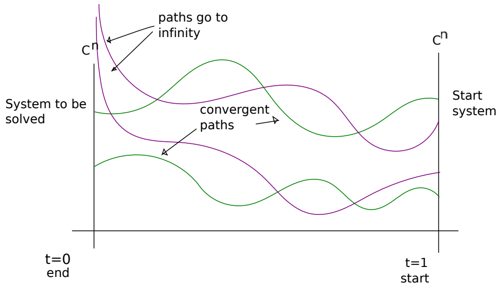
Our objective is to maximize some function Ω of these four workspaces. We used,

$$\Omega = (1-\lambda)\cdot|W_f| + \lambda(|W_1| - |W_\cap|),$$

where λ is a problem-specific weighting factor.

Note: $|W_1| - |W_\cap|$ is a measure of the benefit of having the second robot.

Homotopy Continuation



- Want to solve a square polynomial system (N equations in N unknowns).
- Suppose have solutions to a simple system.
- Deform the known, solved system into unsolved system, forming *solution curves* or *paths*.

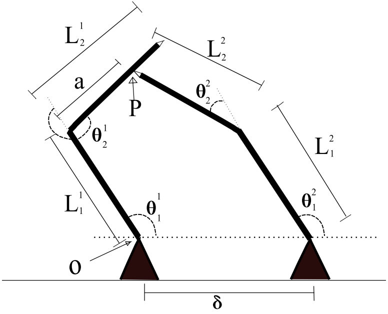
Bertini

The *Bertini* software package: a sophisticated tool for solving polynomial systems with homotopy continuation.
Some benefits and features include:

- Arbitrarily high accuracy,
- Positive-dimensional component detection and (complex) sampling,
- Parallel version with MPI,
- Free to use.

2-Joint Planar Robot, Step 1

- Both robots identical, with all links of length 1. Then $W_1 = W_2$.
- Initial sampling of $[-2\,,2] \times [-2\,,2]$ for computing W_i.
- Inverse kinematics equations is a system of two equations in two variables θ_1, θ_2, which we transform into a system in four unknowns, $s_i = \sin\theta_i$, $c_i = \cos\theta_i$, with $i = 1, 2$.



$$0 = \begin{cases} c_1 c_2 - s_1 s_2 + c_1 - P_x \\ s_1 c_2 + c_1 s_2 + s_1 - P_y \end{cases} . \qquad (1)$$

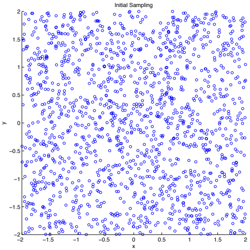
These equations are augmented by trigonometric identities,

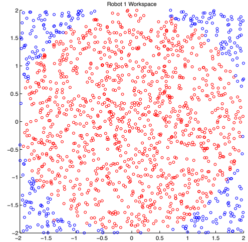
$$c_i^2 + s_i^2 - 1 = 0, \quad i = 1, 2. \qquad (2)$$

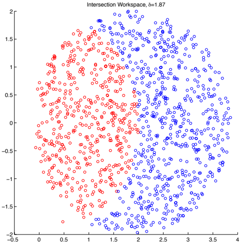
2-Joint Planar Robot, Step 2

Having W_1 compute $W_\cap$.

- Since $W_\cap \subset W_1$, throw out points from original sample which had no solutions.
- $W_\cap = W_\cap(\delta)$, so translate sample for each δ.
- Use same equations (1,2), solve with respect to robot 2.



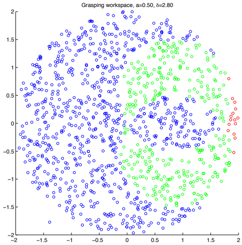




Estimate $|W_i|$, $|W_\cap|$, $|W_f|$, as the fraction of the sample which has real solutions to the inverse kinematics problem.

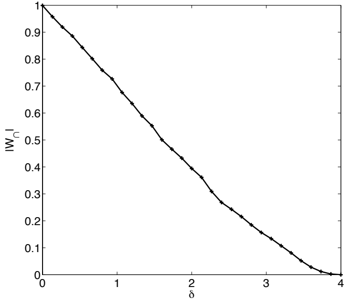
2-Joint Planar Robot, Step 3

- Using $W_f \subset W_1$, solve new equations for grasping configuration.
- Solve for each pair of (δ, a) values.
- W_f system is 8 by 8:

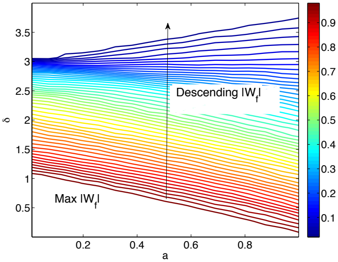


$$0 = \begin{cases} c_1c_2 - s_1s_2 + c_1 - P_x \\ s_1c_2 + c_1s_2 + s_1 - P_y \\ ac_1c_2 - as_1s_2 + c_1 - (c_3c_4 - s_3s_4 + c_3 + \delta) \\ as_1c_2 + ac_1s_2 + s_1 - (s_3c_4 + c_3s_4 + s_3) \\ s_i^2 + c_i^2 - 1 \qquad i = 1, 2, 3, 4. \end{cases}$$

2-Joint Planar Robot, Step 4



(a) $|W_\cap|(\delta)$.

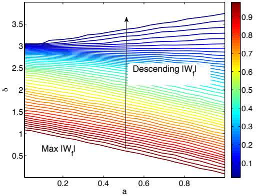


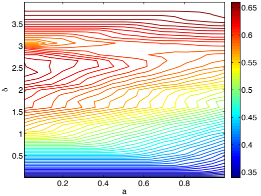
(b) $|W_f|(\delta, a)$.

Plots of intersection workspace and post-failure workspace as functions of δ and a.

$$\Omega = (1-\lambda)\cdot|W_f| + \lambda(|W_1| - |W_\cap|)$$

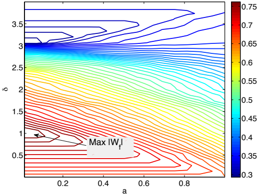
$\lambda = 0.$

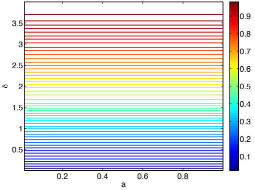




$\lambda = 2/3.$

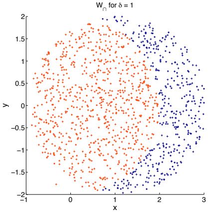
$\lambda = 1/3.$

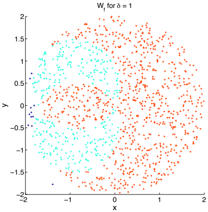




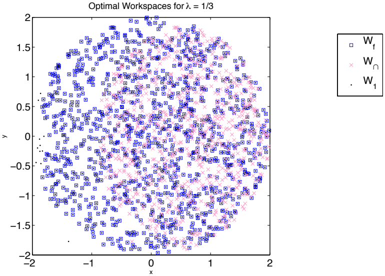
$\lambda = 1.$

Optimal Workspace Demo





Optimal workspaces for
$\lambda = 1/3$.
$(\delta, a) \approx (1, 0)$



Summary

- Optimize placement and grasping location for restoring workspace to broken robot.
- Use *homotopy continuation* via software package *Bertini*.
- Repeatedly solve inverse kinematics problem, for variety of parameter values.
- Optimal configuration for two *two joint rotary robots* depends on user preference – objective function definition, and weighting factors.
- General algorithm presented in this article can be expanded with little difficulty to the following problems:
  - Three dimensional robots,
  - Varying link lengths,

  provided we tackle data storage issues and computational requirements.